terms of a distortion of the configuration coordinate curves, causing a point of near contact between the ground state and the excited state. Then it is clear that F centers will be formed from these α centers preferentially. Firstly, the effective cross section of these α centers will be greater, because once an electron is captured into the excited state, it will not be re-ionized because of the rapid conversion to the ground state. Secondly, such F centers will be more resistant to bleaching than normal F centers. Hence, in the course of bleaching, there will be a tendency for the F centers to become concentrated in these disturbed regions of the crystal. Whether or not this process, if it takes place, has a direct bearing on the formation of M centers is a matter of conjecture. However, it can account qualitatively for the results obtained here.

ACKNOWLEDGMENTS

The authors benefited greatly from discussions with Professor W. D. Compton, particularly on the subject of *M* centers. The evolution of the flash lamp described here was aided by suggestions made by Dr. J. Hartke. Thanks are due to Professor R. J. Maurer and his group for advice and suggestions and for providing a number of materials. Several useful ideas contributed by Dr. A. Gold are gratefully acknowledged.

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Magneto-Optical Oscillations in the Free Carrier and Interband Absorption of Semiconductors

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In the region of the band edge of a degenerate n-type InSb sample, at fixed wavelength, oscillations in the absorption have been observed which are periodic in $1/H$. The period of these oscillations yields information about the Fermi surface. In the region of free carrier absorption oscillations in the absorption have been observed which are periodic in $1/H$, the period yielding the free carrier effective mass. The two measurements hold promise of an optical means of measuring both carrier concentration and effective mass of free carriers in semiconductors.

1. INTRODUCTION

WHILE measuring the free carrier Voigt effect[1] in n-type InSb at room temperature we noticed that in the transverse orientation, there were small oscillations in the free carrier absorption as the magnetic field was swept. Theoretical work by Gurevich *et al.*[2,3] led us to investigate the effect more thoroughly. They showed that an oscillatory effect should be observable in the free carrier absorption and the period of the oscillations would yield the free carrier effective mass. We have observed the effect in n-type InSb both at room temperature and more clearly near liquid-nitrogen temperature.

A similar oscillatory effect is observable in the region of the band edge absorption in degenerate semiconductors. This phenomenon was previously observed by Boyle and Rodgers[4] in bismuth from which they deduced the cross-sectional area of the Fermi surface. We have observed this effect in n-type InSb containing about 1×10^{18} carriers/cm^3 at liquid nitrogen temperature.

2. THEORY

Since the theoretical results given in the two papers of Gurevich *et al.*[2,3] appear to be inconsistent and since no details of the derivations are given, we prefer to base our discussion on an elementary qualitative theory. We restrict our attention to simple parabolic energy bands.

Turning first to interband absorption, we consider the case of direct transitions from magnetic levels of a simple parabolic valence band with maximum at $\mathbf{k}=0$ to magnetic levels of a simple parabolic conduction band with minimum at $\mathbf{k}=0$. The absorption coefficient in intrinsic material shows maxima for photon energies given by[5]

$$\hbar\omega = E_G + \hbar\omega_{c1}(l+\tfrac{1}{2}) + \hbar\omega_{c2}(l+\tfrac{1}{2}), \qquad (1)$$

where ω_{c1} and ω_{c2} refer to the conduction and valence band cyclotron frequencies, respectively, and E_G is the energy gap in zero magnetic field.

Let us now consider n-type extrinsic material with carrier concentration N. For simplicity, we take the

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[1] S. Teitler and E. D. Palik, Phys. Rev. Letters **5**, 546 (1960).

[2] L. E. Gurevich and Z. I. Uritskii, Soviet Phys.—Solid State **1**, 1188 (1960).

[3] L. E. Gurevich, I. P. Ipatova, and Z. I. Uritskii, *Proceedings of the International Conference on Semiconductor Physics, Prague, 1960* (Czechoslovakian Academy of Sciences, Prague, 1961), p. 328.

[4] W. S. Boyle and K. F. Rodgers, Phys. Rev. Letters **2**, 338 (1959).

[5] E. Burstein, G. S. Picus, R. F. Wallis, and F. Blatt, Phys. Rev. **113**, 15 (1959).

temperature to be 0°K. In zero magnetic field the material exhibits a Burstein effect,[6] the absorption coefficient rising sharply at a photon frequency given by

$$\hbar\omega = E_G + E_F[1 + (m_1^*/m_2^*)] \qquad (2)$$

where the Fermi energy E_F is measured relative to the conduction band edge and is given by

$$E_F = (h^2/2m^*)(3N/8\pi)^{2/3}, \qquad (3)$$

where m^* now refers to the conduction band effective mass. Suppose that the photon frequency is now fixed at the value given by Eq. (2) and the external magnetic field is varied. Absorption maxima occur at magnetic fields satisfying the relation

$$E_F = \hbar\omega_c(l + \tfrac{1}{2}) \qquad (4)$$

or

$$l + \tfrac{1}{2} = (E_F m^* c/e\hbar)(1/H) \qquad (5)$$

$$= (\pi c h/e)(3N/8\pi)^{2/3}(1/H). \qquad (6)$$

If the absorption maxima are labeled in succession by numbers $l + \frac{1}{2}$ and plotted against $1/H$, a straight line should result whose slope is $(\pi c h/e)(3N/8\pi)^{2/3}$. Alternatively, the absorption coefficient plotted as a function of $1/H$ is periodic with period P given by

$$P = (e/\pi c h)(8\pi/3N)^{2/3}. \qquad (7)$$

Experimental measurements of the period enable one to obtain a value of the carrier concentration N. Furthermore, if E_F is measured directly using the Burstein effect in zero magnetic field, one can obtain the effective mass m^* by means of Eq. (5).

Let us now consider free carrier absorption. The photon frequency ω is assumed to satisfy the conditions

$$\omega \gg \omega_0, \quad E_F/\hbar, \quad \omega_c, \qquad (8)$$

where ω_0 is a mean phonon frequency, E_F is the Fermi energy, and ω_c is the cyclotron frequency. Furthermore, let the magnetic fields be sufficiently large so that $\hbar\omega_c \simeq E_F$ and the temperature sufficiently low so that $kT \simeq E_F$. Under these conditions only the first few magnetic levels are appreciably occupied.

Free carrier absorption typically arises through second-order processes involving the interaction of the carriers with the radiation and the scattering of the carriers by ionized impurities or by phonons. In the presence of an external magnetic field the scattering processes make possible a breakdown in the usual cyclotron resonance selection rule[7] $\Delta l = \pm 1$ where l is the Landau quantum number for the magnetic levels with energies.

$$E(l,k_z) = \hbar\omega_c(l + \tfrac{1}{2}) + \hbar^2 k_z^2/2m^*. \qquad (9)$$

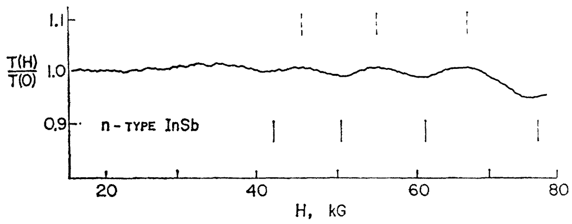

FIG. 1. Oscillations in the interband absorption of InSb near 20°K.

The quantity k_z is the propagation vector component parallel to the magnetic field. When only the first few magnetic levels are occupied, the carriers are concentrated in states near $k_z = 0$, since the density of states is infinite at $k_z = 0$ and decreases monotonically as $|k_z|$ increases. If the final states occur at an energy corresponding to the edge of a magnetic level, the absorption coefficient is expected to be a maximum because of the infinity in the density of final states. By energy conservation these absorption maxima occur at photon frequencies satisfying the equation

$$\hbar\omega = n\hbar\omega_c \pm \hbar\omega_0, \qquad (10)$$

where n is an integer, the plus sign refers to emission of phonons and the minus sign to absorption of phonons. For ionized impurity scattering the term involving ω_0 in Eq. (10) should be omitted. In view of Eq. (8) we shall neglect this term for phonon scattering also.

Equation (10) can be rewritten in the form

$$n = \omega/\omega_c = (\omega m^* c/e)(1/H). \qquad (11)$$

If the absorption maxima for fixed ω and variable H are labeled in succession by integers and these numbers are plotted against $1/H$, a straight line should result whose slope is $\omega m^* c/e$. Alternatively, the absorption coefficient plotted as a function of $1/H$ is periodic with period P given by

$$P = e/\omega m^* c. \qquad (12)$$

The periodic free carrier absorption is to be expected for both polarization cases $E \perp H$ and $E \| H$ where E is the electric vector of the radiation. Experimental meas-

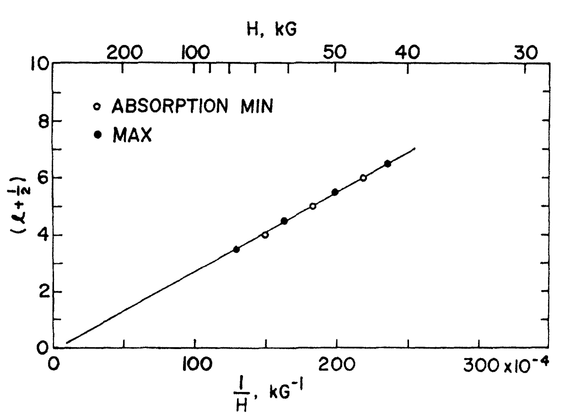

FIG. 2. Extrema in the interband oscillations vs $1/H$ for InSb near 20°K.

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[6] E. Burstein, Phys. Rev. **93**, 632 (1954).

[7] R. F. Wallis, J. Phys. Chem. Solids **4**, 101 (1958).

urements of the period enable one to obtain the effective mass of the carriers.

3. EXPERIMENTAL TECHNIQUES AND RESULTS

The experimental results were obtained using two different Bitter-type, air core solenoidal magnets, one with a 4-in. cylindrical aperture and one with a 1.25-in. cylindrical aperture producing steady fields as high as 80 kG and 150 kG, respectively. A NaCl prism monochromator equipped with a Reeder thermocouple was used for all the work. Samples of n-type InSb for the free carrier measurements were of the order of 1 mm thick, with carrier concentrations in the range 10^{16}–10^{17} at liquid-nitrogen temperature. The sample for the band edge measurements was about 300 μ thick with carrier concentration $\sim 1\times 10^{18}$ cm^{-3}.

For some of the samples, measurements were made for both the longitudinal orientation with direction of propagation parallel to magnetic field and transverse orientation with direction of propagation perpendicular to magnetic field. For the transverse orientation linear polarized radiation produced with an AgCl pile-of-plates polarizer was sometimes used.

Figure 1 shows the transmission near the band edge of a sample containing about 1×10^{18} cm^{-3} as a function of magnetic field. Since there is a Burstein effect, the edge in this sample is at 3.5 μ. Measurements at various wavelengths in the edge indicated no measurable change in period of the oscillations. The amplitude of the oscillations decreased as the measurements were carried to longer wavelengths just outside the absorption edge. Measurements were made using liquid nitrogen and liquid helium in the Dewar. However, with liquid helium, the sample temperature was about 20°K. In Fig. 2 the maxima and minima in transmission have been indexed with an integer or half integer and plotted as a function of $1/H$. The solid line is a least squares fit to the experimental points. The period of the oscillations obtained from the reciprocal slope is 3.57×10^{-6} G^{-1}. Using the formula $P=(e/\pi ch)(8\pi/3N)^{2/3}$, the carrier concentration was found to be 8.5×10^{17} cm^{-3}. This sample has been used in other magneto-optical experiments, namely, magnetoplasma reflection, rotation and ellipticity.[8] These gave a carrier concentration of 1.0×10^{18} in reasonable agreement with the above result. Our present measurements indicate that the straight line of Fig. 2 goes roughly through the origin.

For the InSb sample used, the measured shift in the band edge compared to an intrinsic sample was about 0.11 eV at about 20°K. Equation (2) was used to determine E_F assuming $m_1^*/m_2^*=0.1$. Then Eq. (5) was used to determine m^* from the slope of the $1/H$ vs $(l+\frac{1}{2})$ curve. The result was $m^*/m=0.032$ in reasonable agreement with the value 0.035 observed in other magneto-optical experiments.

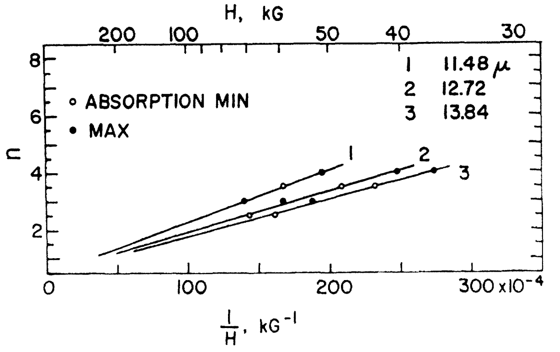

FIG. 3. Extrema in the free carrier oscillations vs $1/H$ for InSb near liquid-nitrogen temperature.

The free carrier oscillations were readily observed at room temperature on several samples with carrier concentration in the range 10^{16} to 10^{17} cm^{-3}. The amplitude of the oscillations increased considerably on cooling to liquid-nitrogen temperature. For one sample the Faraday and Voigt effects yielded $N=1.5\times 10^{16}$ and $m^*/m=0.017$. For this sample in Fig. 3, the maxima and minima are plotted against $1/H$ for several wavelengths. The solid lines are least square fits through the experimental points. The change in period with wavelength is quite noticeable. The period $P=e/(c\omega m^*)$ was used to calculate m^*. The effective masses obtained at various wavelengths are summarized in Table I. The average result was $m^*/m=0.0185$ in reasonable agreement with other magneto-optical measurements.

The low temperature measurements reported in the various figures were made with magnetic fields as high as 80 kG. With increasing field, the oscillations increase in amplitude, but decrease in number for equal magnetic field intervals. Figure 4 shows the free carrier oscillations observed in another InSb sample at room temperature up to 150 kG for the transverse orientation.

At the longer wavelength shown, the onset of very strong absorption due to the ordinary cyclotron resonance appears at the highest fields. At the shorter wavelength, the region of strong cyclotron resonance absorption has not been reached. The effective mass obtained was $m^*/m=0.0187$ which is reasonable for a sample in the range 10^{16} to 10^{17} carriers/cm^3 at room temperature.

TABLE I. Effective masses obtained from free carrier oscillatory absorption in InSb near liquid-nitrogen temperature.

Wavelength λ (μ)	Period P (G^{-1})	m^*/m
14.37	7.35×10^{-6}	0.0182
13.84	7.20	0.0180
13.30	6.55	0.0189
12.72	6.65	0.0180
12.12	6.03	0.0188
11.48	5.55	0.0193
		Av = 0.0185

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[8] E. D. Palik, S. Teitler, B. W. Henvis and R. F. Wallis, *Proceedings of the International Conference on the Physics of Semiconductors*, Exeter, *1962* (to be published).

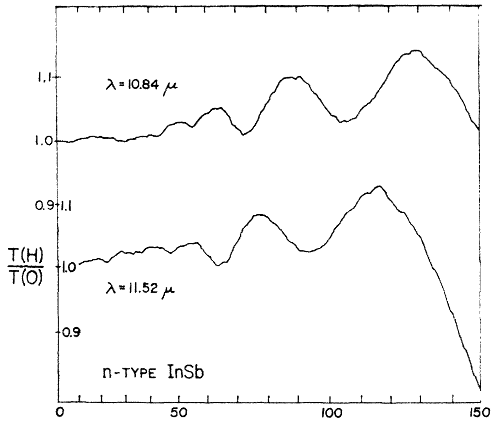

FIG. 4. Oscillations in the free carrier absorption of InSb at room temperature.

Attempts were made to observe free carrier oscillations in the previously discussed sample with carrier concentration 1.0×10^{18} cm^{-3}. The effective mass had been found to be 0.035 from other magneto-optical measurements. The calculated period is quite small, and no distinct oscillations were observed near liquid nitrogen temperature. Further refinement in experimental techniques are underway to allow measurements at liquid He II temperatures with increased spectral resolution.

In the transverse orientation we have made measurements of the oscillations at room temperature and near liquid nitrogen temperature using plane polarized light. At room temperature the amplitudes of the oscillations are of comparable magnitude for the $E \| H$ and $E \perp H$ polarizations. When the temperature is lowered the amplitudes of the $E \perp H$ oscillations increase more rapidly than do those of the $E \| H$ oscillations.

While the greatest amplitude of oscillations is only a few percent in the present experiments, we feel that with improved signal-to-noise ratio in the near infrared and use of liquid-helium temperatures, the oscillations should be observable in other semiconductors such as lead sulfide, where the period will generally be much smaller. The techniques should be useful for determining both carrier concentration and effective mass. Also, for bands with energy ellipsoids, this measurement should give effective mass anisotropy.

4. DISCUSSION

Periodic free carrier absorption in an external magnetic field can arise through any of several mechanisms which lead to a breakdown in the usual cyclotron resonance selection rule $\Delta l = \pm 1$. Besides the second-order scattering mechanism discussed in this paper, one can have a relaxation of the selection rule due to a nonuniform radiation field. This mechanism can become important if the skin depth is small and leads to the Azbel'-Kaner type of cyclotron resonance.[9] In cubic crystals Azbel'-Kaner resonance is observed only in the transverse configuration, i.e., the external magnetic field is parallel to the surface of the sample and perpendicular to the direction of propagation of the radiation. It seems unlikely that the periodic absorption in InSb reported in this paper is due to Azbel'-Kaner resonance, first, because the skin depth is too large and, second, because the oscillations were observed when the radiation propagated parallel to the external magnetic field.

The cyclotron resonance selection rules may also be relaxed by deviations of the energy band from spherical symmetry.[7] For *n*-type InSb such an effect would lead to third harmonics but not second harmonics. No oscillations should be observed for the case $E \| H$. Consequently, the experimentally observed oscillations are probably not due to this mechanism.

Little attention has been devoted in this paper to the phase of the free carrier oscillations. The calculation of the phase is complicated by the necessity of correctly calculating the scattering interaction matrix elements. As pointed out by Cohen and Blount[10] the phase may be strongly affected by the large magnitude of the g factor for conduction electrons in InSb. Experimental measurements of the phase together with a proper theoretical treatment could provide a direct means of determining the g factor.

The change in g factor with energy should give rise to two series of oscillations close together which were not resolved in the present experiments.

The nonparabolic character of the conduction band in InSb should lead to a variation of period with magnetic field. Such an effect has been observed experimentally, when the magnetic field becomes greater than 90 kG, the period decreasing at the higher magnetic fields. In heavily doped samples for which $E_F \gg \omega_c$, nonparabolic effects may be expected to cause some smearing out of the oscillations. This may be at least a partial explanation for the failure to observe oscillations in the free carrier absorption in the 1×10^{18} cm^{-3} sample.

The analysis of the period of the interband absorption neglected certain complications arising from the degeneracy of the valence band in InSb. The existence of heavy hole and light hole magnetic ladders should lead to two periods rather than one. The quantum effects[11] near the valence band edge may cause deviations from periodicity when low quantum number magnetic levels are passing through the Fermi level. The phases of the oscillations will also be affected by the quantum effects. It should be emphasized that the periodic interband absorption discussed here for extrinsic material differs only in certain details from that already extensively investigated in intrinsic material.[5,12] The experiments on

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[9] E. Burstein, P. J. Stiles, D. N. Langenberg, and R. F. Wallis, Phys. Rev. Letters **9**, 260 (1962).

[10] M. H. Cohen and E. I. Blount, Phil. Mag. **5**, 115 (1960).

[11] J. M. Luttinger, Phys. Rev. **102**, 1030 (1956).

[12] L. M. Roth, B. Lax, and S. Zwerdling, Phys. Rev. **114**, 90 (1959).

extrinsic material can be done using relatively thick samples whereas those on intrinsic material require very thin samples. Furthermore, there is no photon frequency in the intrinsic case which can be related to a carrier concentration.

Interference fringes in a plane parallel slab of InSb have been observed to shift in an external magnetic field due to the changes in the free carrier contribution to the index of refraction.[13] The fringe effects are best observed using circular and linear polarization in the longitudinal and transverse orientations, respectively. At fixed wavelength, this shift in fringes with field could produce oscillations in the transmission as the orders of interference passed the wavelength of observation. However, the oscillations would be periodic in *H*. For the particular samples used, the fringes would be about 1 cm^{-1} apart, appreciably less than the spectral resolution of 10 cm^{-1} used. Therefore, the shift of interference fringes is not likely the cause of the observed phenomenon.

A few comments may be worthwhile concerning the relationship of the oscillatory optical absorption phenomena discussed in this paper and other oscillatory effects such as the de Haas-van Alphen effect and the Shubnikov-de Haas effect. The latter two effects depend critically on the existence of a sharp Fermi surface. If the carriers become nondegenerate, oscillations characteristic of these effects disappear. In the optical absorption effects, on the other hand, the basic requirement for oscillations is that the optical radiation excite carriers to final states at an energy coincident with the edge of a well-defined magnetic level. The existence of a sharp Fermi surface is not essential, but does make possible an identification such as Eq. (2) which then leads to the useful Eq. (6). If the relaxation time of the carriers is too short, however, the edges of the magnetic levels will be poorly defined and the oscillations will be smeared out.

Note added in proof. Free carrier oscillations have been observed in 1-mm-thick *n*-type InAs containing about 5×10^{16} carriers $/\text{cm}^3$ at liquid nitrogen temperature. An effective mass ratio of 0.028 was obtained in good agreement with the effective mass obtained from cyclotron resonance measurements in the same magnetic field range 100–150 kG.

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[13] E. D. Palik, Appl. Optics (to be published).

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Exchange Narrowing of *d* Bands in Antiferromagnets*

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The theory of exchange narrowing of *d* bands presented in a previous paper for the ferromagnet is extended to the case of the antiferromagnet. It is shown that the dynamic spin wave-electron interaction gives rise to a narrowing factor $e^{-\zeta}$. ζ depends upon the state of excitation of the spin system and increases with the number of antiferromagnetic magnons and hence with temperature. It is suggested that this effect may contribute to the high resistivity of a large number of antiferromagnetic compounds.

I. INTRODUCTION

In a previous paper[1] (hereafter referred to as I) it was shown that in the tight-binding approximation a *d* electron moving through a ferromagnetic crystal with a Bloch-type wave function would be coupled to the spin-wave system. A semiphenomenological theory describing the interaction of the *d* electron with the spin waves was formulated. The intra-atomic exchange arising in the Hartree-Fock equations was treated as an electron-magnon coupling operator. The coupled Hamiltonian was then separated into effective perturbed electron and spin-wave Hamiltonians. The average effect of the spin waves on electron was shown to result in a dynamic interaction which gave rise to a localization of the electron and a narrowing of the *d* band. The electronic bandwidth was found to depend parametrically upon the state of excitation of the spin system, and decreased with temperature (analogous to the polaron effect).

In this paper we consider the problem of an itinerant *d* electron in an antiferromagnet and show that a similar band narrowing can result. Our principal interest is in the effect of the spin waves on the electronic wave function and bandwidth and we shall not concern

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* Supported by the Air Force Office of Scientific Research.

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[1] T. Wolfram and J. Callaway, Phys. Rev. **127**, 1605 (1962).